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Ning Pei
Iowa State University

Leonard J. Bond
Iowa State University, bondlj@iastate.edu

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Abstract

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Comparison of acoustoelastic Lamb wave propagation in stressed plates for different measurement orientations

Ning Pei and Leonard J. Bond[a)]
Department of Aerospace Engineering and Center for Nondestructive Evaluation, Iowa State University, Ames, Iowa 50011, USA
npei@iastate.edu, bondlj@iastate.edu

Abstract: High order Lamb waves are investigated for the effects of stress on both symmetrical and anti-symmetrical modes in an aluminum plate for wave propagation and load parallel. Data are compared with those for the case of load and measurement axis perpendicular. It is the S_1 mode which exhibits significantly higher sensitivity to stress than other Lamb modes. For aluminum the use of the S_1 mode for stress measurement is found to be about six times more sensitive, than bulk waves, for the load-measurement axes parallel case and this compares with about ten times for the case of load-measurement axes perpendicular.

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1. Introduction

Lamb waves are seeing increasing use in various forms of material characterization, structural health monitoring and nondestructive testing. It was observed by Gandhi *et al.*[1] that although acoustoelastic bulk waves have been quite thoroughly investigated there have been significantly fewer papers that have considered Lamb waves. Their work used both numerical models and experiments to demonstrate how phase and group velocity change, for fundamental Lamb wave modes, with the loading direction. This approach was recently reviewed and extended to higher order Lamb modes by Pei and Bond, who had initially investigated the sensitivity of higher Lamb modes to stresses theoretically[2] for the case where the direction of stress and axis for velocity measurement are perpendicular. The models were then validated with data from recently reported experimental measurements.[3]

In this paper, new data for higher order Lamb mode sensitivity to stress are reported for the case where the load and measurement axis are parallel. Data are compared and contrasted with the case of perpendicular load and measurement axis.[3] The study considered a 1.6 mm thick aluminum plate, with velocity dispersion characteristics reported for the normalized frequency-thickness range from 0 to 20 MHz-mm.

2. Numerical model

The analytical expressions to predict effects on velocity for measurement and load axis perpendicular were reported previously by Gandhi *et al.*[1] and then by Pei and Bond.[2] For the case of measurement and load axis parallel, the form of the equations are generally similar to the perpendicular case so only the differences are reported here. The expression for the initial stress tensor can be written as

$$T = \begin{bmatrix} T_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (1)$$

The complete system of equations[3] is solved, using the boundary conditions for stress free surfaces. Following some manipulation and after a few steps, the dispersion relations for symmetric and anti-symmetric modes are given and these can be written as,[1,3] for symmetric modes,

$$f_s(\omega, c) = D_{11}G_1\cot(\gamma\alpha_1) + D_{13}G_3\cot(\gamma\alpha_3) + D_{15}G_5\cot(\gamma\alpha_5) = 0. \quad (2)$$

And for anti-symmetric modes,

[a)]Author to whom correspondence should be addressed.

Table 1. Aluminum parameters in calculations (Ref. 1).

Aluminum parameters for calculation		
Parameter	Value	Units
λ	54.308	GPa
μ	27.174	GPa
l	−281.5	GPa
m	−339.0	GPa
n	−416.0	GPa
ρ_0	2704	kg/m^2
v_l	6303	m/s
v_t	3102	m/s
Thickness	1.6	mm

$$f_a(\omega, c) = D_{11}G_1 \tan(\gamma\alpha_1) + D_{13}G_3 \tan(\gamma\alpha_3) + D_{15}G_5 \tan(\gamma\alpha_5) = 0. \quad (3)$$

The various constants used to evaluate these equations are given in detail in the supplementary materials.[4] The equations were solved and used to investigate the effects of stress on different Lamb modes in the case where load and measurement axis are parallel and using a MATLAB code, the dispersion curves for the various modes can be obtained. The material parameters used here are the same as those reported previously[1] and the values are given in Table 1.

3. Numerical results

The highest sensitivity of various Lamb modes to load (strain) in the frequency and thickness domain has been seen at 3.0 MHz-mm, near the cut-off frequency.[3] The relative change of group velocity with strain, for the range from 0 to 600 $\mu\varepsilon$, was calculated. These data are then compared with sensitivity for bulk waves for steel[5] and aluminum[6] and the data are shown given in Fig. 1. A difference between the two data sets is that for the perpendicular (B) case the value is positive while for the parallel (A) case the value is negative. This appears to be in agreement with the assumption that tension makes the velocity increase while compression makes the velocity decrease, for the parallel (B) case. For the perpendicular (A) case the opposite is seen. Tension makes the velocity decrease while compression makes the velocity increase. When comparing effects of stress on Lamb waves (configuration A and B) to bulk waves, it is seen that for the S_1 parallel (B) case this Lamb mode is about six times more sensitive than bulk wave and the S_1 mode for the perpendicular (A) is about ten times more sensitive.

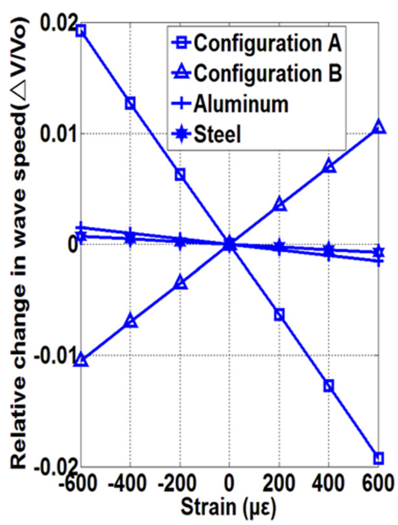

Fig. 1. (Color online) The change of velocity with strain of compressional wave for steel (Ref. 4) and for aluminum (Ref. 5) compared with data for the S_1 mode in aluminum for Configuration A (load and measurement axis perpendicular) and Configuration B (load and measurement axis parallel).

The reason why a certain Lamb mode is much more sensitive than the bulk wave can be explained by considering the dispersion relationship: for the velocity-frequency curve, in terms of the absolute value of the slope this varies for different modes with the change of frequency. For the S_1 mode near the "cut-off" frequency the slope is so steep that this causes a rapid velocity change. For the bulk wave, the slope of the frequency-velocity curve remains nearly constant and exhibits a lower rate of change than the slope of for the S_1 mode.

4. Experiment results

The model data were compared with results obtained using an experimental system reported previously[7] and which was used to obtain data for the case of load and measurement axis perpendicular. In this new work load axis and that for velocity measurement were parallel. The experiment system is shown in schematic form in Fig. 2.

The Lamb waves were generated and received by using piezoelectric transducers on variable-angle Plaxiglas wedges. The transmitted wave is generated using a 30 cycle tone burst provided by a signal generator (Hewlett Packard 33120A, Renton, WA) and a high power amplifier (Model 3100L, Electronic Navigation Industry, Rochester, NY). The received signal is measured with a digital oscilloscope (HDO4022, 200 MHz High Definition Oscilloscope, Teledyne LeCroy, Chestnut Ridge, NY). Samples were aluminum sheets 1.6 mm thick, and the length and width were approximately 50 and 45 cm. The generated strain is monitored by three strain gages with a P3 Strain Indicator and Recorder (Vishary Measurements Group, Inc., Wendell, NC).

5. Data analysis

The effect of load on the velocity of the S_1 Lamb wave mode in the aluminum plate was investigated. The received signal was analyzed using an approach reported previously for the perpendicular (A) case[3] that employed the short-time Fourier Transform (STFT) method,[8] which has been demonstrated to be effective for use in dispersive curve analysis.[9] It has been reported[3] that the arrival time t for the group velocity at specific frequencies can be obtained by determining the magnitude of the coefficients, calculated by STFT method. The arrival time t corresponds to the magnitude of the coefficient. Further detailed discussion for STFT method for group velocity calculation can be found in the paper by Pei and Bond.[3]

For the pair of transducers at a set separation (d) in the assembly the arrival time (t_1) was measured using the STFT method for a certain frequency of S_1 mode. The load was then applied and the new arrival time (t_1') recorded. The transducer separation was increased (or decreased) by a pre-selected increment (d'), and the new arrival time recorded (t_2), and again the load was applied and the arrival time recorded (t_2'). The group velocity can then be calculated using the expression: $Vg = d'/\ (t_2 - t_1)$. The group velocity under loading can also then be expressed as: $Vg_L = d'/\ (t_2' - t_1')$. The time domain data were recorded with loads from 0 to 600$\mu\varepsilon$, with the interval of 100$\mu\varepsilon$. The measurements were each performed three times and data averaged. Strain and time of flight (TOF) data were recorded, their mean ($\bar{x}$) is regarded as a good estimate of the true value, the error bars are expressed as standard error ($S = \sigma/\sqrt{N}$, where σ is the standard deviation and N is the number of the measurement). This approach was used to give an estimate for the error bars reported in the following figures.

A comparison between the velocity changes seen under load between the parallel (case B) and perpendicular (case A) are shown in the following Fig. 3, with the relationship between strain and relative change of velocity at 3.0 MHz-mm. For case B, it is seen that the absolute value of slope of the experiment data is 1.44e–5 $\mu\varepsilon^{-1}$,

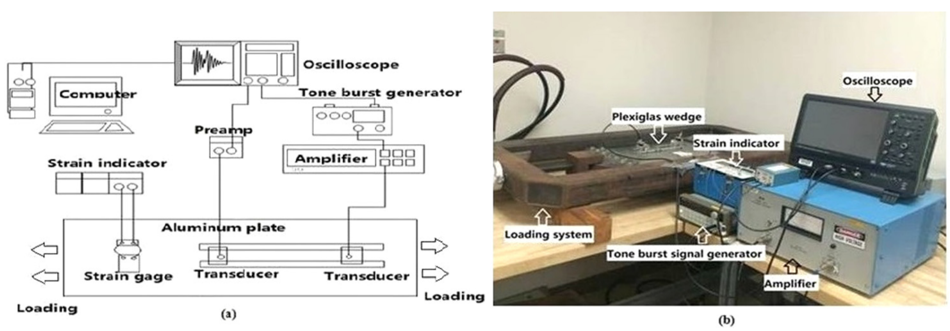

Fig. 2. (Color online) Schematic for experimental system used to measure strain and ultrasonic velocity change under various loads.

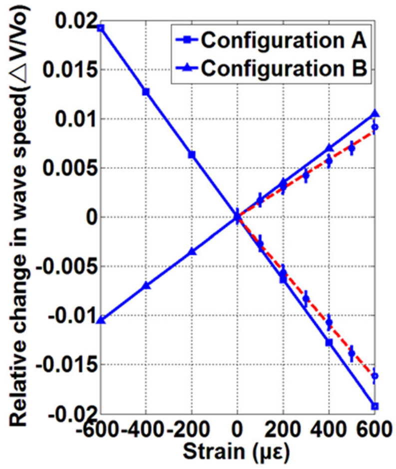

Fig. 3. (Color online) Comparison of experimental and model S_1 velocity change against load (dash line) and numerical model results for cases at 3.0 and 3.015 MHz-mm. aluminum for load and measurement axis perpendicular (configuration A) and load and measurement axis parallel (configuration B).

which is a little less than that for the model result data (1.67e–5 $\mu\varepsilon^{-1}$). The reason for the difference is that in the experiment the frequency and mode generated were not the "cut-off" values. The dash line bar represents the error bar for the results, and this was small ($\pm 7.5e-4$) as it is shown in the figure. As was also found previously for the case of measurement axis and load perpendicular (case A) when the S_1 mode velocity was investigated it was also found to be slightly less sensitive to load than that predicted for the cut-off at 3.0 MHz-mm. The reason is believed to be the same in that in the experiment the frequency and modes generated were not exactly the "cut-off" values. The experimental and model data for these two cases are in good agreement when values assumed to be higher than the cut-off, as shown with the red dashed lines for case at 3.01 MHz-mm rather than 3.0 MHz-mm.

6. Conclusions

New data for the cases of the effect of stress on both symmetrical and anti-symmetrical higher order Lamb modes in an aluminum plate for the configuration of wave propagation and load parallel is given. Data are compared with those for load and measurement axis perpendicular. In terms of effect on velocity it is the S_1 mode which exhibits significantly higher sensitivity to stress than other Lamb modes. For aluminum the use of the S_1 mode for stress measurement is found to be about six times more sensitive than bulk waves for the parallel case and this compares with about ten times for the perpendicular case.

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